THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA, VADODARA

Ph. D. ENTRANCE TEST (PET) – 7th August 2022

Signature of Invigilator

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Paper - II
Mathematical Sciences
(22/28)

Roll. No.

Maximum Marks: 50 **No. Of Printed Pages: 8**

Instruction for the Candidate:

1. This paper consists of **FIFTY (50)** multiple choice type questions. Each Question carries **ONE (1)** mark.
2. There is no Negative Marking for Wrong Answer.
3. A separate OMR Answer Sheet has been provided to answer questions. Your answers will be evaluated based on your response in the OMR Sheet only. No credit will be given for any answering made in question booklet.
4. Defective question booklet or OMR if noticed may immediately replace by the concerned invigilator.
5. Write roll number, subject code, booklet type, category and other information correctly in the OMR Sheet else your OMR Sheet will not be evaluated by machine.
6. Select most appropriate answer to the question and darken appropriate oval on the OMR answer sheet, with black / blue ball pen only. DO NOT USE PENCIL for darkening. In case of over writing on any answer, the same will be treated as invalid. Each question has exactly one correct answer and has four alternative responses (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example:Ⓐ ⬤ Ⓒ Ⓓ where (B) is correct response.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
9. Calculators, Log tables any other calculating devices, mobiles, slide rule, text manuals etc are NOT allowed in the examination hall. If any of above is seized from the candidates during examination time; he/ she will be immediately debarred from the examination and corresponding disciplinary action will be initiated by the Center Supervisor as deemed fit.
10. DO NOT FOLD or TEAR OMR Answer sheet as machine will not be able to recognize torn or folded OMR Answer sheet.
11. **You have to return the OMR Answer Sheet to the invigilator at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet on conclusion of examination.

Paper - II

Mathematical Sciences (22/28)

Note: This paper contains **FIFTY (50)** multiple-choice questions. Each Question carries **ONE (1)** mark.

(1) A set A is said to be countable if there exists a function f

(11) If every element of a group G is its own inverse, then G is

(A) Cyclic group (B) Finite group

(C) Infinite group (D) Abelian group

(12) Which sentence is true?

(A) Set of all matrices forms a group under multiplication.

(B) Set of all rational negative numbers forms a group under multiplication.

(C) Set of all non-singular matrices forms a group under multiplication.

(D) Both (b) and (c)

(13) The set of all real numbers under the usual multiplication operation is not a group since

(A) multiplication is not a binary operation.

(B) multiplication is not associative.

(C) identity element does not exist.

(D) zero has no inverse

(14) All integral domain S is

(A) field when S is finite. (B) always a field.

(C) never field. (D) none of these.

(15) Topology is derived from two greek words topos and logos, where the meaning of topos is _____________.

(A) Study (B) Geometry (C) Surface (D) None of these

(16) The following is true for the following partial differential equation used in nonlinear Mechanics known as the Korteweg-de Vries equation. $\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} \quad 6w\frac{\partial w}{\partial x} = 0.$

(A) linear; 3rd order. (B) nonlinear; 3rd order.

(C) linear; 1st order. (D) nonlinear; 1st order.

(17) Which one of the following is essential for solving Partial Differential equations.

(A) Boundary conditions. (B) Algebraic equations.

(C) Mathematical Model. (D) none of these..

(18) The value of grad(a) where ‘a’ is constant is equal to

(A) 1 (B) 0 (C) 1 (D) Cannot be determined

(19) Given the following table:

x	0	1	2	4
y	-5	-6	5	111

What is the value of $\frac{dy}{dx}$ at $x = 1$?

(A) 2 (B) 3 (C) 4 (D) 8

(20) If we apply Euler-Cauchy method to solve the initial value problem $\frac{dy}{dx} = x + y,\ y(0) = 0,$ taking $\quad = 0.2$ then we get:

(A) $y_{n+1} = y_n + 0.2(x_n + y_n)$ (B) $y_{n+1} = y_n + 0.2(x_{n-1} + y_{n-1})$

(C) $y_{n+1} = y_{n-1} + 0.2(x_n + y_n)$ (D) $y_{n+1} = y_{n-1} + (0.2)^n(x_n + y_n)$

(21) The integral of function $f(x) = 3|x \quad 1| + 2|x + 2|$ over the interval [-1,1] is computed using Trapezoidal rule with partition of step size (0.2). The difference between the computed value and actual value is:

(A) 0.2 (B) 0 (C) $\quad 0.2$ (D) 0.4

(22) Inversion maps circle $|z \quad 1 + i| = \overline{2}$ onto:

(A) line of w-plane passing through origin.

(B) line of w-plane not passing through origin.

(C) circle of w-plane passing through origin.

(D) circle of w-plane not passing through origin.

(23) Function $f(z) = e^x(siny + icosy)$ is:

(A) analytic everywhere in z-plane except at $z = 0$.

(B) analytic everywhere in z-plane.

(C) only analytic at $z = 0$.

(D) nowhere analytic in z-plane.

(24) The radius of convergence of power series expansion of $\frac{2z+3}{z+1}$ in the powers of $(z \quad 1)$ is:

(A) (B) 1 (C) 2 (D) 0

(25) If $r = \{z \quad |z| = 3\}$, then the value of $_r \frac{cos(\pi z)}{(z-2)z^2} dz$?

(A) 0 (B) $\quad 2\pi i$ (C) $\frac{-3\pi i}{2}$ (D) $\frac{-\pi i}{2}$

(26) The Set $B = \{z/\ |Re(z)| > 2\}$ is:

(A) neither domain nor bounded (B) domain and bounded

(C) unbounded and domain (D) bounded but not domain

(27) $f(z) = sinz$ is:

(A) Bounded function, for all z, in z-plane

(B) Nowhere Conformal in z-plane

(C) Unbounded function, for all z, in z-plane

(D) Conformal everywhere in z-plane

(28) Harmonic conjugate function of $u(x, y) = y^3 \quad 3x^2y$ is:

(A) $v(x, y) = x^3 \quad 3y^2 + C$ (B) $v(x, y) = 3xy^2 \quad x^3 + C$

(C) $v(x, y) = 3xy^2 \quad y^3 + C$ (D) $v(x, y) = x^3 \quad 3xy^2 + C$

(29) The singular solution of the differential equation $(y\ )^3 \quad 4xyy' + 8y^2 = 0$, is:

(A) $27y + 4x^3 = 0$ (B) $27y \quad 4x^3 = 0$

(C) $27y + 2x^3 = 0$ (D) $27y \quad 2x^3 = 0$

(30) For the differential equation $y\frac{dy}{dx} + x = 0$ which of the following is a solution?

(A) $x^2 + y^2 = c\ , x \quad R$ (B) $y = \quad \overline{c \quad x^2}, \qquad \overline{c} < x < \quad \overline{c}$

(C) $y = \quad \overline{c \quad x^2}, \qquad \overline{c} \quad x \quad \overline{c}$ (D) $y = \quad \overline{c \quad x^2}, \qquad \overline{c} \quad x \quad \overline{c}$

(31) Let $y(x)$ be the solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = sinx,\ x \quad R$. Then $\lim_{x \quad \infty} y(x)$:

(A) equals zero (B) equals 1

(C) equals -1 (D) does not exist

(32) Which of the following differential equation has an unbounded solution for $x > 0$

(A) $y'' + \omega^2 y = cos(\omega x)$ (B) $y'' + \omega^2 y = e^{-\omega x}$

(C) $y'' + \omega^2 y = cos(2\omega x)$ (D) $y'' + \omega^2 y = sin(2\omega x)$

(33) The differential equation $(x^3 + y^3)dx + 3y^2 x dy = 0$ is:

(A) exact, homogeneous and linear (B) exact, non-homogeneous and linear

(C) exact, homogeneous and non-linear (D) not exact, homogeneous and non-linear

(34) General solution of $x^2\frac{d^2y}{dx^2} \quad \frac{5}{2}x\frac{dy}{dx} \quad 2y = 0$, is:

(A) $y = Ax^{\frac{1}{2}} + Bx^4$ (B) $y = Ax^{\frac{-1}{2}} + Bx^4$

(C) $y = Ax^{\frac{1}{2}} + Bx^2$ (D) $y = Ax^{\frac{-1}{2}} + Bx^2$

(35) The sequence $\left\{(999)^{1/n}\right\}$

(A) converges to 0 (B) converges to 1

(C) diverges to (D) converges to 999.

(36) Which of the following functions is not uniformly continuous on $(0,1)$?

(A) $x \sin\frac{1}{x}$ (B) x^2 (C) $\frac{\sin x}{x}$ (D) $e^x \cos\frac{1}{x}$.

(37) If $[x]$ denotes the greatest integer less than, or equal to, x, then the limit $\lim_{x \quad 0}\frac{[x]}{x}$

(A) does not exist (B) is equal to 1 (C) is equal to 0 (D) is equal to $\quad 1$.

(38) Let $f: [0,1] \qquad$ be defined as $f(0) = 0$ and $f(x) = \frac{1}{n}$ if $\frac{1}{n+1} < x \quad \frac{1}{n}$ for $n = 1,2, \ldots$. Then on $[0,1]$, f is

(A) not Riemann integrable (B) Riemann integrable but not continuous

(C) continuous but not monotonic (D) Riemann integrable as well as continuous.

(39) The linear fractional transformation that maps the points $z_1 = 2, z_2 = i, z_3 = \quad 2$ on to the points $w_1 = 1, w_2 = i, w_3 = \quad 1$ is given by

(A) $w = \frac{z+2i}{z+4}$ (B) $w = \frac{3z-2i}{z+6}$ (C) $w = \frac{z+5i}{z+3}$ (D) $w = \frac{3z+2i}{iz+6}$

(40) If $p(x)$ is a differentiable function and $p(x) > 0$, then the Wronskian of solutions $W(x)$ of the equation $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + q(x)y = 0$ is

(A) $W(x) = \frac{c}{p(x)}$, c being a constant (C) $W(x) = c\ exp[\quad q(x)dx]$, c being a constant

(B) $W(x) = p(x)\frac{dq(x)}{dx} + q(x)\frac{dp(x)}{dx}$ (D) $W(x) = 0$.

(41) Eigen values of a real symmetric matrix are always a

(A) positive real number (B) negative real number

(C) purely imaginary number (D)real number

(42) Which of the following is not a field?

(A) $C[0,1]/I$, where $I = \{f \quad C[0,1] | f\left(\frac{1}{3}\right) = 0\}$ (B) /

(C) $_{11}/(x^2 + 1)$ (D) /

(43) In the Hilbert space ^{2}, consider $S = \{(i, 1)\}$. Then the set $S \quad S^{\perp}$ is equal to

(A) (B) $\{(0,0)\}$ (C) ^{2} (D)S

(44) Consider the Hilbert space l_2 and its two subsets $M = \{x = \{x(n)\}: x(2n) = 0, n \quad \}$,

$N = \left\{x = \{x(n)\}: x(2n) = \frac{x(2n-1)}{2n}, n \quad \right\}$. Then which one of the following is true?

(A)M is not a subspace (B)N is not a subspace

(C)M is not closed (D)M and N are closed subspaces.

(45) The intersection of the subgroups ($, +$) and ($, +$) of the group ($, +$) is the subgroup

(A) ($, +$) (B) ($, +$) (C) ($, +$) (D) ($, +$)

(46) A real-valued function defined on an interval $[a, b]$ need not be differentiable almost everywhere if it is

(A) absolutely continuous (B) continuous

(C) of bounded variation (D) monotonic.

(47) If $f: [a, b]$ is Lebesgue integrable and if $\int_a^b f(t)dt = 0$, then

(A) $f(t) = 0$ for all t in $[a, b]$ (B) $f(t) \quad 0$ for all t in $[a, b]$.

(C) $f(t) = 0$ for almost all t in $[a, b]$ (D) $f(t)$ may not be equal to 0 for any t in $[a, b]$.

(48) A measure space $(X, \mathfrak{B}, \mu)$ is complete if $\mathfrak{B}$ contains all subsets of sets of

(A) measure 1 (B) finite measure (C) measure zero (D) infinite measure.

(49) If a function $f: [a, b]$ is of bounded variation, and P, N, and T are respectively the positive, negative, and total variation of f on $[a, b]$, then

(A) $P = N + T$ (B) $T = P + N$ (C) $N = P + T$ (D) $P + T + N = 0$.

(50) If φ is a convex function on (a, b) then

(A) φ is differentiable on (a, b)

(B) $\varphi''(x) \quad 0$ for all $x \quad (a, b)$

(C) $\varphi''(x) \quad 0$ for all $x \quad (a, b)$

(D) the right-hand derivative of φ exists at every point of (a, b).

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Rough Work: