THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA, VADODARA

Ph. D. ENTRANCE TEST (PET) 2023

Signature of Invigilator

Roll. No.

Paper - II

Mathematical Sciences

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Maximum Marks: 50 **No. Of Printed Pages: 8**

Instruction for the Candidate:

1. This paper consists of **FIFTY (50)** multiple choice type questions. Each Question carries **ONE (1)** mark.
2. There is no Negative Marking for Wrong Answer.
3. A separate OMR Answer Sheet has been provided to answer questions. Your answers will be evaluated based on your response in the OMR Sheet only. No credit will be given for any answering made in question booklet.
4. Defective question booklet or OMR if noticed may immediately replace by the concerned invigilator.
5. Write roll number, subject code, booklet type, category and other information correctly in the OMR Sheet else your OMR Sheet will not be evaluated by machine.
6. Select most appropriate answer to the question and darken appropriate oval on the OMR answer sheet, with black / blue ball pen only. DO NOT USE PENCIL for darkening. In case of over writing on any answer, the same will be treated as invalid. Each question has exactly one correct answer and has four alternative responses (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example: Ⓐ ● Ⓒ Ⓓ where (B) is correct response.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
9. Calculators, Log tables any other calculating devices, mobiles, slide rule, text manuals etc are NOT allowed in the examination hall. If any of above is seized from the candidates during examination time; he/ she will be immediately debarred from the examination and corresponding disciplinary action will be initiated by the Center Supervisor as deemed fit.
10. DO NOT FOLD or TEAR OMR Answer sheet as machine will not be able to recognize torn or folded OMR Answer sheet.
11. **You have to return the OMR Answer Sheet to the invigilator at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet on conclusion of examination.

Paper - II

Mathematical Sciences

Note: This paper contains **FIFTY (50)** multiple-choice questions. Each Question carries **ONE (1)** mark.

1	Let $f_n(x) = \frac{x}{1+n^2x^2}$, $x \in [0, 1]$, $n = 1,2, \ldots$. Consider the series of functions $\sum_{n=1}^{\infty} f_n$. Then which one of the following statements is FALSE?
	(A) $\sum_{n=1}^{\infty} f_n$ converges uniformly on $[\frac{1}{4}, \frac{1}{2}]$ (B) $\sum_{n=1}^{\infty} f_n$ converges uniformly on $[\frac{1}{6}, \frac{1}{3}]$ (C) $\sum_{n=1}^{\infty} f_n$ converges uniformly on $[\frac{1}{2}, 1]$ (D) $\sum_{n=1}^{\infty} f_n$ converges uniformly on $[0,1]$
2	Let $f_n(x) = (-1)^n \frac{x^2+n}{n^2}$, $x \in \mathbb{R}$, $n = 1,2, \ldots$. Then which one of the following is TRUE?
	(A) $\sum_{n=1}^{\infty} f_n$ diverges on $\mathbb{R}$ (B) $\sum_{n=1}^{\infty} f_n$ converges absolutely on $\mathbb{R}$ (C) $\sum_{n=1}^{\infty} f_n$ converges uniformly on $[-A, A]$ for all $A > 0$. (D) $\sum_{n=1}^{\infty} f_n\,(0)$ is divergent
3	Let f be Riemann integrable on $[a, b]$, $A = \left\{\int_a^b \psi(x)dx: f \leq \psi, \psi \text{ is a step function}\right\}$, and $B = \left\{\int_a^b \phi(x)dx: \phi \leq f, \phi \text{ is a step function}\right\}$. Then value of the upper Riemann integral of f over $[a, b]$ is given by
	(A) $\inf A$ (B) $\sup A$ (C) $\inf B$ (D) $\sup B$
4	Let f be measurable function defined on a measurable set E. If $\int_E f = 0$, then which one of the following is TRUE?
	(A) $\int_E f^+ = 0$ (B) $\int_E f^- = 0$ (C) $f = 0$ $a.e.$ on E (D) $m(\{x \in E: f(x) \neq 0\})$ may be positive
5	Consider the following two statements: (I) If f is the pointwise limit of $f_n(x) = \begin{cases} 1, & \text{if } n \leq x < n \\ 0, & \text{if } 0 \leq x < 1 \end{cases}$, $n \in \mathbb{N}$. Then $\int_0^\infty f < \liminf_{n\to\infty} \int_0^\infty f_n$; (II) Let f and g be non negative measureable functions defined on a measureable set E, Then $\int_E f - g = \int_E f - \int_E g$. Then which one of the given options is correct? (A) (I) is TRUE but (II) is FALSE (B) both (I) and (II) are TRUE (C) both (I) and (II) are FALSE (D) (II) is TRUE but (I) is FALSE
6	Consider the group $(\mathbb{Z}, +)$ and its subgroup $H = 3\mathbb{Z}$, then the number of distinct cosets of H in G will be
	(A) 3 (B) 2 (C) 1 (D) 4
7	If $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 8 & 6 & 11 & 10 & 9 & 12 & 5 & 3 & 7 & 2 & 1 & 4 \end{pmatrix}$, then $o(\tau)$ is given by
	(A) 60 (B) 40

	(C) 20 (D) 12
8	Suppose V is a real vector space and U and $W$are subspaces of V. Consider the following two statements: (I) $U \cap W$ is a subspace of V; (II) $U \cup W$ is a subspace of V if $U \subset W$. Select the correct alternative
	(A) (I) is TRUE but (II) is FALSE (B) both (I) and (II) are TRUE (C) both (I) and (II) are FALSE (D) (II) is TRUE but (I) is FALSE
9	Suppose V is a real vector space and S is a non empty subset of V which does not contain the zero vector. Which one of the following statements is FALSE? ($[S]$ = span of S)
	(A) $[S]$ is a subspace of V (B) $S \subset [S]$ (C) $\vec{0} \in [S]$ (D) $S = [S]$
10	Inversion maps circle $\|z-1-i\| = 1$ onto:
	(A) line of w-plane passing through origin (B) line of w-plane not passing through origin (C) circle of w-plane passing through origin (D) circle of w-plane not passing through origin
11	Function $f(z) = e^x(cosy - isiny)$ is:
	(A) analytic everywhere in z-plane except at $z = 0$ (B) analytic everywhere in z-plane (C) only analytic at $z = 0$ (D) nowhere analytic in z-plane
12	The radius of convergence of power series expansion of $\frac{2z+3}{2z-z^2}$ in the powers of $(z+2i)$ is
	(A) ∞ (B) 1 (C) 2 (D) 0
13	If $r = \{z : \|z\| = 3\}$, then the value of $\int_r \frac{cos(\pi z)}{(z-2)z^2} dz$?
	(A) 0 (B) $-2\pi i$ (C) $\frac{-3\pi i}{2}$ (D) $\frac{-\pi i}{2}$
14	The Set $B = \{z \,/\, \|Re(z)\| = \|z\|\}$ is:
	(A) neither domain nor bounded (B) domain and bounded (C) unbounded and domain (D) bounded but not domain
15	$f(z) = \sin(\bar{z})$ is :
	(A)Bounded function, for all z, in z-plane (B) Nowhere Conformal in z-lane (C) Analytic everywhere in z-plane (D) Conformal everywhere in z-plane

16	Harmonic conjugate function of $u(x,y) = 3xy^2 - 4xy - x^3$ is:
	(A) $v(x,y) = 2(x^2-y^2)+3xy^2-y^3+C$ (B) $v(x,y) = y^3+3xy^2-2(x^2-y^2)+C$ (C) $v(x,y)$ *does not exist* (D) $v(x,y) = 2(x^2-y^2)-3xy^2+y^3+C$
17	If a function $f(z) = u(x,y)+iv(x,y)$ is analytic in a domain D, then its component functions u and v are ________________________________.
	(A) not harmonic in D. (B) Not analytic in D. (C) Analytic but not harmonic in D. (D) Harmonic in D.
18	The value of $\int_C \frac{(3z+2)^2}{z(z-1)(2z+5)} dz$, where C is the positively oriented circle $\lvert z \rvert = 3$, is
	(A) $9\pi i$ (B) $-3\pi i$ (C) $2\pi i$ (D) 0
19	Given the following table: x: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 y: 30.13, 31.62, 32.87, 33.64, 33.95, 33.81, 33.24 What is the value of $\frac{dy}{dx}$ at $x = 0.3$?
	(A) 3.33 (B) 1.33 (C) 5.33 (D) 6.33
20	Given the initial value problem $\frac{dy}{dx} = x^2 + y$, $y(1) = 2$, taking $h = 0.25$, applying Euler's method estimation of $y(2)$ = ______
	(A) 9.90626 (B) 10.90626 (C) 7.90626 (D) 8.90626
21	The integral of function $f(x) = 3\lvert x-1 \rvert + 2\lvert x+2 \rvert$ over the interval [-1,1] is computed using Trapezoidal rule with partition of step size (0.2). The difference between the computed value and actual value is :
	(A) 0.2 (B) 0 (C) -0.2 (D) 0.4
22	The singular solution of the differential equation $y = xy' - (y')^2$ is:
	(A) $y = \frac{x^3}{3}$ (B) $y = \frac{x^2}{4}$ (C) $y = \frac{x^4}{4}$ (D) $y = \frac{x^2}{2}$
23	For the differential equation $y\frac{dy}{dx} + x = 0$ which of the following is a solution?
	(A) $x^2 + y^2 = c, x \in R$ (B) $y = \sqrt{c-x^2}, \quad -\sqrt{c} < x < \sqrt{c}$ (C) $y = -\sqrt{c-x^2}, \quad -\sqrt{c} \le x \le \sqrt{c}$ (D) $y = \sqrt{c-x^2}, \quad -\sqrt{c} \le x \le \sqrt{c}$

24	Let $y(x)$ be the solution of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=sinx,\ x\in R$. Then $\lim_{x\to\infty} y(x)$:
	(A) equals zero (B) equals 1 (C) equals -1 (D) does not exist
25	Which of the following differential equation has an unbounded solution for $x>0$?
	(A) $y" + \omega^2 y = cos(wx)$ (B) $y" + \omega^2 y = e^{-wx}$ (C) $y" + \omega^2 y = cos(2wx)$ (D)(D) $y" + \omega^2 y = sin(2wx)$
26	The differential equation $(y^3-x^3)dy = 3x^2ydx$ is:
	(A) exact, homogeneous and linear (B) exact, non-homogeneous and linear (C) exact, homogeneous and non-linear (D) inexact, homogeneous and non-linear
27	General solution of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$, is:
	(A) $y=Ax^{\frac{1}{2}}+Bx^2$ (B) $y=Ax^{-1}+Bx^2$ (C) $y=Ax^2+Bx^{-2}$ (D) $y=Ax^{\frac{-1}{2}}+Bx^2$
28	The directional derivative at $x=(2,0,-1)$ for $f(x)=6y^2-18yz-6zx+2xy-7x+5y-6z-4$ in the direction of the vector $Y=(1,1,1)'$ is
	(A) $\frac{8}{3}$ (B) $\frac{8}{\sqrt{3}}$ (C) $-\frac{8}{3}$ (D) $-\frac{8}{\sqrt{3}}$
29	Solution of the NLPP : Min $z=2x_1^2-24x_1+2x_2^2+8x_2+2x_3^2+12x_3+200$, Subject to constraints: $x_1+x_2+x_3=1,\ \ x_1,x_2,x_3\ge 0$ is ________________.
	(A) $(2,3,6)$ (B) $(3,2,6)$ (C) $(6,2,3)$ (D) $(6,3,2)$
30	$A=\begin{pmatrix}3&1&2\\1&5&0\\2&0&2\end{pmatrix}$ is
	(A) Positive definite (B) Positive semidefinite (C) Negative definite (D) Negative semidefinite
31	The IVP dy/dx+y^2=x; y(0)=1 is
	(A) linear (B) nonlinear (C) linear with fixed constants (D) undeterminable to be linear or nonlinear
32	$y''-4y'+8y=0$ for $y(0)=1$ and $y'(0)=2$
	(A) $y=4e^{2t}\cos(2t)-4e^{2t}\sin(2t)$ (B) $y=e^{2t}\cos(2t)$ (C) $y=4e^{2t}-2e^{-2t}$) (D) $y=e^{2t}\cos(2t)-e^{2t}\sin(2t)$

33	$y'' + \lambda y = 0$, $y(-L) - y(L) = 0$, $y'(-L) - y'(L) = 0$
	(A) Not Sturm Liouville problem (B) Sturm Liouville problem (C) Sturm Liouville problem and boundary conditions are separated (D) None of the above
34	The partial differential equation $\partial^2 y / \partial t^2 = 6\partial^2 y / \partial x^2$ is classified as
	(A) Elliptic (B) Parabolic (C) Hyperbolic (D) None of the above Solution
35	When solving a 1-Dimensional heat equation $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$ using a variable separable method, we get the solution if _______________
	(A) k is positive, where k is a constant which we are assuming during Separable variable process (B) k is negative, where k is a constant which we are assuming during Separable variable process (C) k is 0, where k is a constant which we are assuming during Separable variable process (D) k can be anything, where k is a constant which we are assuming during Separable variable process
36	$u_x = cu + d(x, y)$, $u(0, y) = y$ is
	(A) Cauchy problem (B) Neumann Problem (C) Robin Problem (D) None of the above
37	The Lagrangian equations of motion are ____________ order differential equations.
	(A) first (B) second (C) zero (D) forth
38	The shape of the Normal Curve is ____________
	(A) Bell Shaped (B)spherical shape (C)Circular (D) Spiked
39	Determine the value of the functional S[y] = $\int_0^1 y'(x)^2 dx$ for the functions y(x) = x
	(A) 0 (B)1 (C)2 (D)none of the above
40	The Boundary value problem has
	(A) Always Unique Solution (B) Always existence of solution (C) Infinitely many solution (D) None of the above
41	The Greens function for y "(x) +y= x, y(0) = 0, y(1) = 0.
	(A) Exists. (B) Does not exist (C) Sometimes Exist (D) None of the above

42	The Boundary conditions occurs in in BVP ut=u_xx, u(0,t)=0; u(l,t)=0, u(x,0)=f(x) is
	(A)Dirichlet condition (B)Neumann Condition (C)Robin Condition (D)None of the above
43	For which of the following distributions mean and variance are equal?
	(A) Normal Distribution (B) Poisson Distribution (C) Binomial Distribution (D) Negative Binomial Distribution
44	The solution of $u_x{}^2 + y\, u_y - u = 0$ using Charpits method is
	(A)Possible (B)Not possible (C)Possible under certain condition (D)None of the above
45	The calculus of variations is concerned
	(A) Increasing and decreasing of function (B) With the maxima or minima of functional. (C) Boundedness of functions (D) None of The above
46	A set of linear equations is represented by the matrix equation Ax = b. The necessary condition for the existence of a solution for this system is
	(A)A must be invertible (B)b must be linearly depended on the columns of A (C)b must be linearly independent of the columns of A (D)None of these
47	Who introduced the term matrix?
	(A) James Sylvester (B) Arthur Cayley (C) Girolamo Cardano (D) Paul Erdos
48	Consider f : R → R. Which of the following is not a linear map?
	(A) f(x) = x (B) $f(x) = x^2$ (C) f(x) = 3x (D) f(x) = 0
49	The residues at the pole $z = 0$ of the function $\frac{cot\pi z}{(z-a)^2}$ is
	(A) $-\pi \sin^2 \pi a$ (B) $-\pi cosec^2 \pi a$ (C) $\pi cos^2 \pi a$ (D) $\pi sec^2 \pi a$
50	The radius of convergence of power series $\sum_{n=0}^{\infty} \frac{z^n}{n}$ is
	(A) 1 (B) 2 (C) 3 (D) 4

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Rough Work: