THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA, VADODARA

Ph. D. ENTRANCE TEST (PET) 2023

Signature of Invigilator

Paper - II
Statistics

Roll. No. | | | | | | |

Maximum Marks: 50 **No. Of Printed Pages: 8**

Instruction for the Candidate:

1. This paper consists of **FIFTY (50)** multiple choice type questions. Each Question carries **ONE (1)** mark.
2. There is no Negative Marking for Wrong Answer.
3. A separate OMR Answer Sheet has been provided to answer questions. Your answers will be evaluated based on your response in the OMR Sheet only. No credit will be given for any answering made in question booklet.
4. Defective question booklet or OMR if noticed may immediately replace by the concerned invigilator.
5. Write roll number, subject code, booklet type, category and other information correctly in the OMR Sheet else your OMR Sheet will not be evaluated by machine.
6. Select most appropriate answer to the question and darken appropriate oval on the OMR answer sheet, with black / blue ball pen only. DO NOT USE PENCIL for darkening. In case of over writing on any answer, the same will be treated as invalid. Each question has exactly one correct answer and has four alternative responses (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example: Ⓐ ● Ⓒ Ⓓ where (B) is correct response.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
9. Calculators, Log tables any other calculating devices, mobiles, slide rule, text manuals etc are NOT allowed in the examination hall. If any of above is seized from the candidates during examination time; he/ she will be immediately debarred from the examination and corresponding disciplinary action will be initiated by the Center Supervisor as deemed fit.
10. DO NOT FOLD or TEAR OMR Answer sheet as machine will not be able to recognize torn or folded OMR Answer sheet.
11. **You have to return the OMR Answer Sheet to the invigilator at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet on conclusion of examination.

Paper - II
Statistics

Note: This paper contains **FIFTY (50)** multiple-choice questions. Each Question carries **ONE (1)** mark.

01) Given that *a* and *b* are real numbers, let f(*a*,*b*) = *ab* and let g(*a*) = a^2+2. What is f(3,g(3))?
A) 27
B) 30
C) 33
D) 38

02) A bag contains 6 blue and 8 yellow balls. Find the probability of drawing two yellow balls in succession, when the ball drawn is not replaced.
A) 4/49
B) 4/13
C) 24/91
D) 45/91

03) Given $y_0 = -12$, $y_2 = 4$, $y_3 = 6$, and $y_4 = 12$. What is the value of y_1 using Newton's formula
A) 0
B) 1
C) 2
D) 3

04) If $A = \begin{bmatrix} 3 & -7 \\ -4 & 1 \end{bmatrix}$, then the determinant of $A^3 + 3A^2 + 12A$ is
A) 29000
B) -29500
C) 30000
D) -39500

05) _____ is the systematic approach to the development, operation and maintenance of software.
A) Hardware Engineering
B) Process Engineering
C) Software Engineering
D) System Engineering

06) The order and degree of the differential equation $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5$ is
A) order 1, degree 1
B) order 1, degree 2
C) order 2, degree 1
D) order 2, degree 2

07) For the series, 29, 12, 26, 19, 24, 36, 21, 33, and 35; the value of the first quartile is
A) 20
B) 21
C) 22
D) 23

08) Let N(t) be a Poisson process with constant intensity function λ on R. What is the covariance of N(s) and N(t)?
A) λs, if s<t
B) λ(t-s), if s<t
C) λ(s-t), if (t<s)
D) λ(s+t)

09) The joint pmf of (X, Y) is given by
P(x,y) = cxy^2, for x=1,2,3 and y=1,2.
What is the value of c?
A) 1/10
B) 1/20
C) 1/30
D) 1/40

10) Consider the following statements:
I. If i is inessential and i leads to j then j is essential
II. If i is recurrent, j is transient then i do not lead to j
Which of the following are correct?
A) Only I is correct
B) Only II is correct
C) Both I and II are correct
D) Neither I nor II is correct

11) P = $\begin{bmatrix} .5 & .3 & .2 \\ .2 & .5 & .3 \\ .2 & .8 & 0 \end{bmatrix}$. What is the value of f_{11}^3 ?
A) 0.234
B) 0.166
C) 0.080
D) 0.036

12) The Chapman-Kolmogorov equation of a particular stochastic process is given as follows:

$$P_k(h) = \begin{cases} 1 - \mu_n(h) + o(h), k = 0 \\ \mu_n(h) + o(h), k = 1 \\ o(h), k \geq 2 \end{cases}$$

If μ_n is the rate of arrival, then the process is
A) Poisson Process
B) Pure birth process
C) Pure death process
D) Linear process

13) For an M/M/1 queuing system, the arrival and departure rate are respectively μ and θ. What is the variance of the steady state distribution?
A) $\frac{\mu\theta}{(\mu-\theta)^2}$
B) $\frac{(\mu\theta)^2}{(\mu-\theta)}$
C) $\frac{\mu\theta}{\mu-\theta}$
D) $\frac{(\mu\theta)^2}{(\mu-\theta)^2}$

14) A planning method designed to show the tasks associated with a project, the personnel responsible for completing the tasks and the timelines allowed Insurance against disturbances which may or may not occur.
   A) Pareto chart
   B) Flow chart
   C) Gantt chart
   D) Process map

15) Industrial experimentation is applicable to
   A) Permanent problems
   B) Occasional problems
   C) Chronic type of problem
   D) Instant problems

16) Which is not a part of Markov decision process (MDP)
   A) A set of finite states
   B) A set of finite actions
   C) A set of Rewards
   D) A set of risks

17) Let the random variable X follows a Poisson distribution with parameter Θ the then unbiased estimator of $e^{-2\Theta}$ is
   A) 1^x
   B) -1^x
   C) 1^{2x}
   D) 1^{-2x}

18) Let $X_1, X_2.....X_n$ be a random sample from $N(\mu,\sigma^2)$, σ^2 being known then the C.R. lower bound to the unbiased estimator of μ will be
   A) n/σ^2
   B) $- n/\sigma^2$
   C) (c) σ^2/n
   D) $- \sigma^2/n$

19) Consider a single observation from the rectangular distribution $dF=dx$, $\Theta \leq x \leq \Theta+1$
Then X will be
   A) a sufficient statistic for Θ
   B) a sufficient statistic and complete statistic for Θ
   C) a sufficient statistic but not a complete statistic for Θ
   D) a complete statistic for Θ.

20) Let $X_1, X_2.....X_n$ be a random sample from $N(\mu,\sigma^2)$, σ^2 being known the for testing the Hypothesis H_0: $\mu = \mu_0$ against the alternative H_1: $\mu \neq \mu_0$ then the L.R.T. will be
   A) a MP test
   B) a UMP test
   C) an UMPU test
   D) a UMPI test

21) In Bayesian inference set up having the sample information from a probability distribution $f(x; \Theta)$ having a proper prior distribution $\Pi(\Theta)$ in order to obtain the Bayes estimator when the over estimation and under estimation are not of equal consequences the appropriate loss function to be chosen is
   A) an squared error loss function
   B) an absolute error loss function
   C) an asymmetric loss function
   D) Weighted squared error loss function.

22) In order to test the hypothesis that the number of male child born is same as the number of female child in a hospital, an appropriate test would be carried out using
   A) Sign test
   B) Two sample Wilcoxon test
   C) Two sample Kolmogorov-Smirnov test
   D) Wilcox on-signed rank test

23) Consider a decision problem with the sample space having two observations $\{X_1, X_2\}$ and the action space having three actions $\{a_1,a_2,a_3\}$ then how many non-randomized decision rules will have 1:1 mapping
   A) 9
   B) 6
   C) 12
   D) 3

24) In general if the density function of a multivariate normal density is written as
$f(X) = k\, e^{-1/2} (X-B)' A (X-B)$ where X and B are the vectors of order 'p' and A is positive definite matrix then the probability density function of X is defined when
   A) $(X-B)' A (X-B) > 0$
   B) $(X-B)' A (X-B) < 0$
   C) $(X-B)' A (X-B) \geq 0$
   D) $(X-B)' A (X-B) \leq 0$

25) Let the sample variance–covariance matrix 'A' be distributed as $W_p(n,\sum)$ and L be an arbitrary vector, then the distribution of $L'AL$ will be
   A) chi-square with 'p' degrees of freedom
   B) chi-square with 'n' degrees of freedom
   C) σ^2_L chi-square with 'n' degrees of freedom
   D) σ^2_L chi-square with 'p' degrees of freedom where $\sigma^2_L = L'\sum L$.

26) Let X is a random vector having the components X_1 as the height of father, X_2 as the height of mother, X_3 as the height of 1st son, and X_4 as the height of second son. In order to calculate the correlation between the height of parents and the progeny (son's) an appropriate method would be to calculate
A) Multiple correlation
B) Partial correlation
C) Canonical correlation
D) Serial correlation

27) Given the assumptions of the CLRM, the least square estimates posses some optimum properties given by the Gauss-Markov theorem. Which of these statements is NOT a part of the theorem?
A) The estimator is linear function of a random variable
B) The average value of the estimator is zero
C) The estimator ha a minimum variance
D) the estimator is unbiased.

28) Consider a clinical trial to assess the effect of drug, diet & exercise on certain disease. The researchers may be interested in getting an answer to the question: 'Is there any difference between the different diets?' Since the different diets may be large, and it may not be possible to conduct such study by taking sample outputs of all different diets. Therefore, keeping this and other related problems in mind a random experiment is performed. Then the sample observations can be well represented by
A) a three-way fixed effects model
B) a three-way mixed effects model
C) a three-way random effects model
D) a three-way random effects model with interactions

29) The F-test in most of the cases will reject the hypothesis that the partial slope coefficients are simultaneously zero. This happens when
A) Multicollinearity is present
B) Multicollinearity is absent
C) heteroscedastcity is present
D) heteroscedastcity is absent.

30) Given model A: $Y_i = \gamma_1 + \gamma_2 X_{2i} + ui$ and model-B: $Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + ui$. Which one of the following statements is true?
A) Model A and B are nested models
B) Model A is nested in model B
C) Model B is nested in model A
D) Model A and B are non- nested models.

31) When the supply of tomatoes or onions reacts to the price with a lag of one time period due to gestation period in production, such a phenomenon is referred to
A) Lag phenomenon
B) Cobweb phenomenon
C) Inertia
D) Business cycle.

32) A random variable X has the following probability distribution:

X	1	2	3
P(X=x)	1/4	2/4	1/4

What is the value of E[X(X-1)]?
A) 5/2
B) 5/4
C) 7/2
D) 7/4

33) If the Durbin-Watson 'd' statistic is found to be equal to zero, this means that the first-order autocorrelation is
A) perfectly positive
B) perfectly negative
C) zero
D) imperfect negative correlation.

34) Let $X_1, X_2, ..., X_n$ be a random sample from b(1;p) then the C.R. lower bound for 'p' will be
A) p(1-p)
B) p(1-p)/n
C) p^2 (1-p)/n
D) $p(1-p^2)/n$

35) Read the following statements carefully and mark the correct choice:
1. Statement 1: Replication is the repetition of an experimental condition so that the variability associated with the phenomenon can be estimated.
2. Statement 2: Randomization is a way when there may be a factor that might be impacting your process but, you are interested in neutralizing that effect.
3. Statement 3: Blocking is the process of assigning the various levels of the Investigated factors to the experimental units in a random fashion.

A) Statement 1 and 2 is correct
B) Only statement 1 is correct
C) Statement 2 and 3 is correct
D) Only statement 2 is correct

36) For the following design which of the statement is true?

Blocks	Treatments
1	1,3,5
2	2,4,6
3	3,5,7
4	4,6,8
5	5,7,1
6	6,8,2
7	7,1,3
8	8,2,4

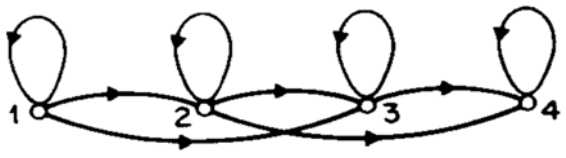
A) Design is equireplicate, proper and orthogonal
B) Design is equireplicate, proper and non orthogonal
C) Design is complete, proper and orthogonal
D) Design is complete, proper and orthogonal

37) Following table is the example of testing of Ignition coil to determine if it will withstand salt water. Find out which one is the design factor (D) and noise factor (N).

Factor	Low level	High level	Factor type
Housing Material	Polyethylene	Polypropylen	?
Water temperature	5ºC	15ºC	?
Seal Thickness	0.02”	0.03”	?
Water Pressure	10 PSI	20 PSI	?
Exposure Time	1 hr	5 hrs	?

A) D, N, D, N, N
B) N, D, N, D, D
C) D, D, N, D, N
D) N, N, D, N, D

38) The following HMM is



A) 4 state ergodic model
B) 4 state left right model
C) 4 state parallel model
D) None of these

39) In the usual notation proper orthogonal transformation is given by:

A) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

B) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

C) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -cos\theta & sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

D) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -cos\theta & sin\theta \\ sin\theta & -cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

40) Let $f(x) = 1/x^2$ on the interval [1,4] and let P={1,2,3,4} be the partition of [1,4] then in the usual notation, oscillatory sum W(f,P) is given by_____
A) 135/144
B) 136/144
C) 62/144
D) none of these

41) Two statements S1 and S2 are given below:

S1: In any metric space X, every convergent sequence is a Cauchy sequence.

S2: In R^k, every Cauchy sequence converges.

Which of the following are correct?
A) Only S1 is correct
B) Only S2 is correct
C) Both S1 and S2 are correct
D) Neither S1 nor S2 is correct

42) A newspaper boy buys papers for Rs. 4 each and sells them for Rs. 6 each. The unsold newspapers cannot be returned. The daily demand for the newspapers follow the distribution as shown in the following table. If each day’s demand is independent of previous day’s demand, how many papers should be ordered each day?

Demand	5	6	7	8	9	10	11
Probability	0.05	0.15	0.4	0.2	0.1	0.05	0.05

A) 5
B) 7
C) 8
D) 9

43) Customers arrive at the first class ticket counter of a theatre at the rate of 12 per hour. There is one clerk serving the customers at the rate of 30 per hour. What is the probability that there is no customer at the booking counter?
A) 0.8
B) 1.0
C) 0.9
D) 0.6

44) The scalar λ is a characteristic root of the matrix A if:
   A) (A- λI) is non singular
   B) (A- λI) is singular
   C) A is non singular
   D) A is singular

45) In CUSUM chart, what does the cumulative sum represent?
   A) The cumulative sum of the process mean
   B) The cumulative sum of the process standard deviation
   C) The cumulative sum of the deviations from the target value
   D) The cumulative sum of the process range

46) Which statistical measure is typically used to define the acceptance quality level (AQL)?
   A) Mean of the population
   B) Median of the population
   C) Standard deviation of the population
   D) Proportion of defective items in the population

47) The reliability of a system over a time interval is calculated as:
   A) The integral of the hazard rate function over that interval.
   B) The derivative of the hazard rate functions over that interval.
   C) The product of the hazard rate function and the time interval.
   D) The inverse of the hazard rate function over that interval.

48) Which of the following statements is true regarding the hazard rate function?
   A) It is always decreasing over time.
   B) It is always increasing over time.
   C) It can be increasing, decreasing, or constant over time.
   D) It is constant for all time.

49) Which one of the following statement is true?
   A) A case is right censored when time of the failure is known only to have occurred after time t.
   B) A case is left censored when time of the failure is known only to have occurred after time t.
   C) In failure censoring with replacement failed units are not replaced.
   D) In type-I censoring the experiment is terminated before a fixed time t.

50) The hazard rate function of exponential distribution with mean λ is given by
   A) $e^{-\lambda t}$
   B) $\lambda e^{-\lambda t}$
   C) λ/t
   D) $\frac{1}{\lambda}$

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Rough Work: