The Maharaja Sayajirao University of Baroda

Ph. D. Entrance Test

Statistics

Syllabus

The syllabus for Ph.D. Entrance test for the subject of Statistics is divided into 8 units having equal weightage as shown below.

Unit – 1: Mathematics

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system, supremum, infimum; Metric spaces, compactness, connectedness.

Convergence of sequences and series, limsup, liminf; Bolzano Weierstrass theorem, Sequences and series of functions, uniform convergence; Continuity, uniform continuity, differentiability, mean value theorem; Functions of several variables, directional derivative, partial derivative; Riemann integral, Improper Integrals; Monotonic functions, types of discontinuity.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension; Algebra of matrices, rank and determinant of matrices; Eigenvalues and eigenvectors.; matrix representation of linear transformations; orthonormal basis; Quadratic forms, classification of quadratic forms

Algebra: Permutations, combinations, Fields, finite fields, field extensions, Galois Theory.

Numerical Analysis: Errors in numeric computations, error analysis; Numerical solutions of algebraic equations- Method of iteration and Newton-Raphson method, Solution of systems of linear equations - Gauss elimination and Gauss-Seidel methods, Interpolation, Numerical integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods; Triangularization of matrices, matrix decompositions.

Unit – 2: Probability

Descriptive statistics, exploratory data analysis

Sample space, discrete probability, independent events, Bayes' theorem. Random variables and probability distributions; expectation and moments. Independent random variables, marginal and conditional distributions. Distribution function, pmf, pdf, moment generating function, Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

Unit – 3: Statistical Inference

Methods of estimation, properties of estimators, sufficiency, Cramer-Rao inequality, Rao-Blackwell and Lehman-Scheffe theorems, UMVU estimation, confidence intervals; Asymptotic efficiency, CAN

estimators, methods of estimation in large samples; Tests of hypotheses: most powerful and uniformly most powerful tests, unbiased tests, similar tests, tests with Neyman structure, construction of UMPU tests, Sequential probability ratio Tests, Invariant tests; likelihood ratio tests, large sample tests; Inference for parameters of multivariate normal distribution, Analysis of categorical data; Elementary Bayesian inference.

Nonparametric tests- one sample, two samples (paired and unpaired), k sample problems, test for independence, tests of randomness.

Decision theory – action space, decision space, loss functions, optimum decision rules, admissibility and completeness, randomized and non-randomized decision rules, Bayes' rules, invariant decision rules.

Unit – 4: Stochastic Modelling

Standard discrete and continuous univariate distributions, sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.

Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms, partial and multiple correlation coefficients, Canonical correlation; Principle component analysis, Discriminant analysis, cluster analysis.

Lifetime distributions – Exponential, Weibull, and Gamma distributions, Extreme value distributions.

Stochastic processes, Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Random walk, Gambler's ruin problem, Branching processes, Poisson process, birth-and-death process, renewal processes; wide-sense stationary processes.

Linear models - Gauss-Markov models, error and estimation spaces, Lease Squares estimation, best linear unbiased estimators, confidence intervals, tests for linear hypotheses. Analysis of variance and covariance. Fixed, random, and mixed effects models, estimation of variance components; diagnostics for linear models.

Regression models, Ridge regression, logistic regression.

Unit – 5: Industrial Statistics

Quality concepts, Statistical process control, Control charts, analysis of patterns on control charts, CUSUM control charts, EWMA control charts, process capability indices; Statistical Quality control – Acceptance sampling plans – by attributes, by variables.

Six-sigma methodology – DMAIC improvement cycle, quality function deployment, FMEA process, process Capability and assessment of Sigma level, identification, organizing, verification and validation of causes, Designing experiments for quality improvement; Taguchi Methods of Parameter Design and Tolerance Analysis

Reliability concepts and measures, reliability of coherent systems, bounds on system reliability; Failure rates, hazard function, notions of ageing, univariate and bivariate shock models, inference using censored sample for different life testing models; Accelerated life testing, Reliability – series, parallel systems, k-out-of n systems.

Unit – 6: Operations Research

Mathematical programming - Linear programming problem: simplex methods, duality, complementary slackness theorem and dual simplex method, sensitivity analysis; Integer programming: cutting plane method, branch and bound technique; dynamic programming; non-linear programming, Kuhn-Tukker conditions and their applications.

Inventory models – deterministic models with and without shortages, stochastic models: single period and two-period models with setup cost.

Queueing theory – Markovian queuing models and Steady-state solutions: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1; models with state dependent arrival and service rates

Unit – 7: Artificial Intelligence

Artificial Intelligence and Machine Learning, Supervised and Unsupervised learning, Classification and regression problems, Feature selection and feature extraction, Model selection, regularization, and cross-validation, Assessment of model accuracy; Model improvement – bagging and boosting.

Pattern recognition problem, density estimation, Kernel density estimation, Bayesian belief networks, Markov Chain Monte Carlo method, classification models, Moment invariants and their applications, Pattern recognition for sequential data – Markov models, Hidden Markov models.

Unit – 8: Sampling, Design of Experiments, and Econometrics

Simple random sampling, stratified sampling and systematic sampling. Varying probability sampling, Horvitz-Thompson estimator, Probability proportional to size sampling, estimation for PPS sampling, two-stage sampling; Use of supplementary information for estimation - Ratio and regression methods.

Completely randomized designs, Block designs – RBD, LSD, BIBD, PBIBD; Connectedness, balanced ness and orthogonality of block designs, 2^k factorial experiments: confounding, construction of block designs; intra-block and inter-block analysis, response surface designs.

Detection of multicollinearity, heteroscedasticity, and autocorrelation, their consequences, and remedial measures; Econometric modelling.